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SOME PRACTICAL ASPECTS OF THE LOT PLOT SAMPLING ACCEPTANCE PLAN

BY

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TECHNICAL REPORT NO. 19

OCTOBER 29, 1954

PREPARED UNDER CONTRACT N6onr-25126
(NR-042-002)

FOR

OFFICE OF NAVAL RESEARCH

APPLIED MATHEMATICS AND STATISTICS LABORATORY

STANFORD UNIVERSITY

STANFORD, CALIFORNIA

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1.0 Introduction

Few sampling acceptance plans have been adopted and used with such apparent success in so short a period of time by as many companies and yet been subject to as much controversial discussion as has the Hamilton Standard Lot Plot plan. The applications of this plan have been declared both successful and unsuccessful in strikingly similar situations by different users. It is subject to criticism for being too strict on one hand and being unreliable on the other. It has been praised for being conservative, yet this conservatism, it is claimed, has resulted in unnecessary screening of acceptable lots.

The purpose of this report is to examine critically the Lot Plot plan and to attempt to evaluate the practical aspects of it.* Since the basic plan, as proposed by Mr. Shainin, has been altered to some extent by almost every user, an item by item evaluation will not be practical. This report, however, will describe some of the current uses and modifications, present both favorable and unfavorable aspects, (as discovered by industrial users) and draw some conclusions regarding the possible future uses.

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* For an analysis of the statistical and theoretical aspects of the plan, see Technical Report No. 18, "Some Theoretical Aspects of the Lot Plot Sampling Inspection Plan", by Lincoln E. Moses.

2.0 Uses

2.1 Original Purpose of Plan

The original purpose of the plan was defined by Dorian Shainin in his statement of the problem [2]. "The problem, briefly, was how to improve .... upon the conventional 100% inspection of purchased material of all sorts that would be received from vendors in varying lot sizes at irregular intervals." The weaknesses of the usual acceptance procedures based on attribute sampling plans as well as those of 100% inspection were recognized by Mr. Shainin. As a manufacturer of airplane propellers, he was dealing with parts of a critical nature and it was extremely important that any acceptance plan provide good protection against the acceptance of sub-standard lots. The plan was designed, therefore, to make maximum use of the information available from a relatively small sample inspected by variables. In addition to the use of the usual statistics, estimate of the lot mean and estimate of the lot standard deviation, the plan made use of a frequency distribution of the sample results. This frequency distribution was important since it was used to estimate the shape of the distribution of the lot from which the sample was drawn and to establish the method by which the lot standard deviation would be estimated. A brief review of the steps in the plan is worthwhile:

1. Select a random sample of 50 pieces, in groups of five, and record the measured quality for each item on a histogram of 7 to 16 cells.
2. Determine the average range of each subgroup of five and record.

3. Classify the histogram by examination as being one of the 11 standard types.*

4. Follow the instructions for the specific type selected to estimate the lot standard deviation and establish the lot limits.

5. Compare the lot limits with the specification limits and accept the lot if the lot limits are within the specification limits.

6. If the lot limits (either or both) are outside of the specification limits, compute the estimated per cent defective in the lot, and hold the lot for Materials Review Board.

The information obtained is, therefore, for the purpose of answering certain questions regarding the lot:

a. Is the lot acceptable? or

b. What percentage of the lot fails to meet specifications and what are the probable limits of variation for the lot?

Clearly, this is more information than ordinarily provided by attribute sampling inspection. It is also the same information provided by 100% inspection and at much less inspection cost. There is no doubt that such information is valuable both from the viewpoint of building up a quality history regarding the vendor and of providing intelligent decisions on the disposal of rejected lots. The real question is,

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* See Figure 1.

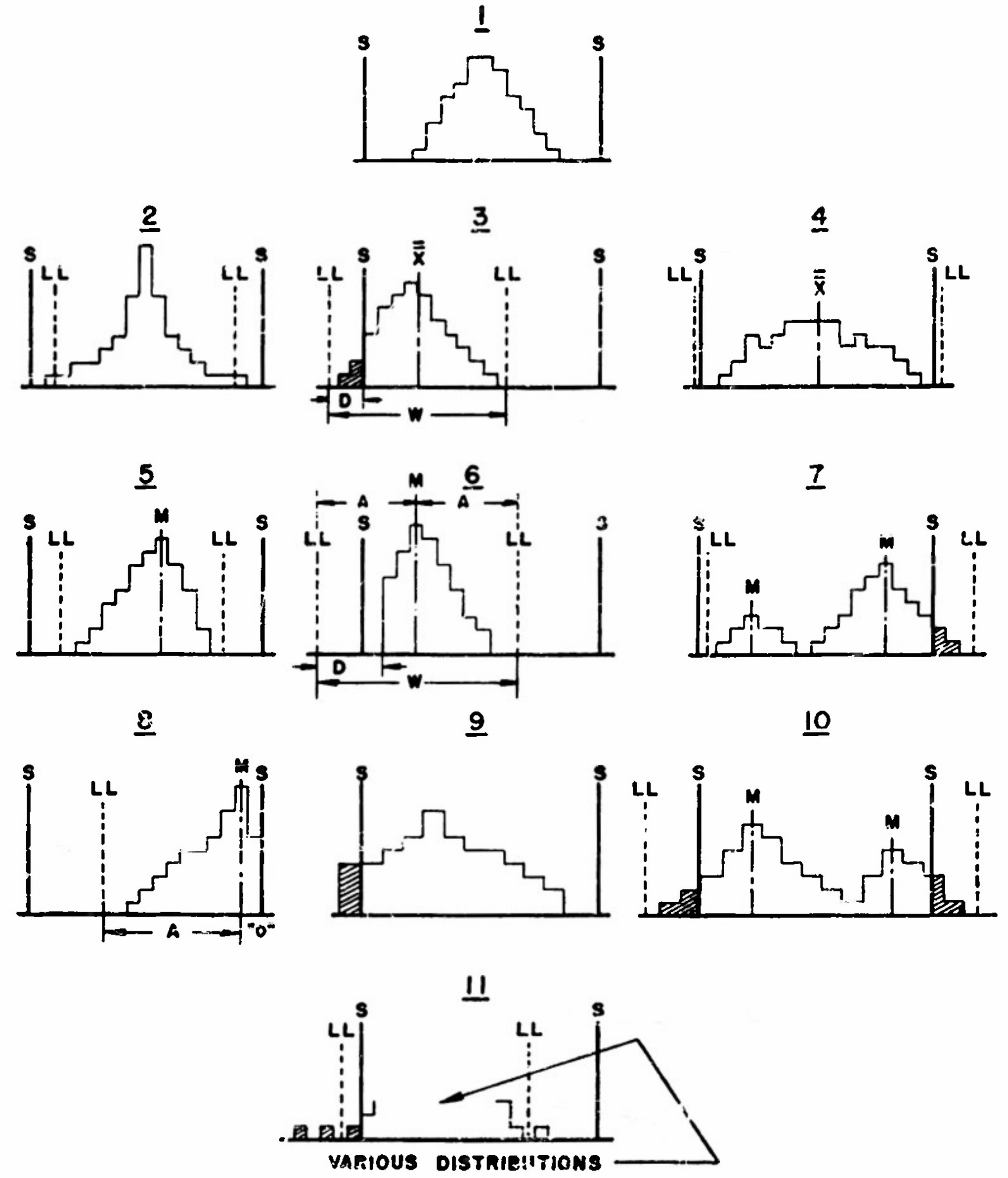


Figure 1 —Eleven Typical Types of Lot Plots

LEGEND

S = specification limit; LL = lot limit.

$\bar{\bar{X}}$ = ave. of subgroup averages; M = mode.
"O"=zero, "full indicator reading."
A = distance in cells from mode to lot limit from Charts C_1 or C_2.
D = distance in cells from spec. limit or cut-off point, as case may be, to lot limit.
W = base width, or distance in cells from one lot limit to the other.

Reproduced by permission of the author and the editor from Dorian Shainin, "The Hamilton Standard Lot Plot Method of Acceptance Sampling by Variables," Industrial Quality Control, Vol. VII, No. 1, July 1950, pp. 15-34.

"Does the Lot Plot method provide reliable estimates of these statistics at less cost than they can be obtained by other methods?"

2.2 What does Lot Plot Actually Replace?

In order to answer this and other questions regarding Lot Plot, Professors Grant and Ireson visited more than sixteen companies that have been using Lot Plot, four Air Force Procurement Offices, and talked with several persons who have been interested in the growth of the method. A substantial part of the following data was accumulated in these visits.

It was found, in general, that the Lot Plot method has been used to replace either 100% inspection or attribute sampling plans (such as Dodge- Romig, Jan-Std-105, Appendix X and Mil-Std-105A) for acceptance purposes. This was true in connection with some parts in every company visited. Some of the uses discovered were of interdepartmental nature and others were for a sort of control on processes, such as heat treatment. More will be said later about these special uses.

It should be noted that many of the so-called 100% inspections that were replaced were neither 100% inspection nor 100% effective. In one instance, incoming shipments of sheet steel, brass or other sheet stock were given a 100% inspection in that the thickness of each sheet was measured at one point, near the edge or end. The sheets were then assorted into separate piles according to this thickness reading. It seems obvious that such a plan is not really 100% inspection since the variation in the thickness of each sheet is not determined, and the total variation in thickness of the sheets in any stack of sorted sheets is likely to be greater than the nominal tolerances assigned to that stack. With the introduction of the Lot Plot plan, a random sample of 50 sheets was drawn from a shipment believed to have been produced in one continuous run. The measurement of each sheet was plotted and the probable variation limits computed. The entire shipment was then assigned to a particular classification by thickness, as 0.026 ± - .002". The basic variation of the entire shipment became the basis for classification and the advantages of such a procedure lie in the fact that no attempt is made to sort within the shipment. Any properly applied variables sampling plan would have provided these advantages.

In other instances the 100% inspection was accomplished by very rapid inspection using go and not-go gauges. It is a known fact that the effectiveness of such rapid attribute inspection is usually fairly low. Good parts are rejected and bad parts are accepted, with the result that 100% inspection is frequently almost worthless. (See paragraph 4.0). The quality level of the accepted product frequently is no better than the incoming lot quality. Clearly, a plan that will provide reliable information about the per cent defective in the incoming lot at considerably less cost is more desirable. In case the incoming quality is as good as the product accepted by the 100% inspection method, the lot might just as well be accepted without the additional cost. If the lot is considerably worse than the acceptable level of defects, the lot will be rejected and subsequently screened at the vendor's expense. Confidence in the Lot Plot plan as a substitute for 100% inspection stems from the fact that the accepted lots were no worse than the product accepted by the former method and the inspection cost had been reduced substantially. A properly selected attributes acceptance plan would probably have produced equally good results, but the users were disillusioned regarding the protection gained by 100% inspection.

The Lot Plot plan, as an example of variables acceptance plans, possesses a number of advantages over attributes plans. In many cases the cost of inspecting fifty pieces by variables is less than the cost inspecting a much larger number by attributes. According to the Operating Characteristic Curve derived by Mr. Shainin for the Lot Plot a matching plan from Mil-Std-105A would require 750 pieces to be inspected by attributes. This is based upon the assumption that both plans must have an AQL value of 0.035% defective. Conversely, by the same standard of matching, there is no Lot Plot plan that would match Mil-Std-105A plans of a higher AQL. This partially accounts for the criticism (discussed later in paragraph 3.0) that the Lot Plot is too strict.

An associated advantage of the Lot Plot stems from the fact that the inspector is less likely to "flinch" on a part that is on the borderline between acceptable and non-acceptable quality. In attribute sampling the inspector is well aware of the situation where one more rejected part will result in the rejection of the lot. There is a tendency for him to call a borderline, or slightly sub-standard, part good under these circumstances that he would ordinarily call defective. In using a variables plan, the inspector is measuring the part and cannot know that a certain measurement will result in the rejection of the lot. If there is a question about it, the inspector may not be informed of the specification limits for the part until after the measurements are completed.

2.3 The Inspector's Part in Lot Plot.

The duties of the inspector under the Lot Plot plan do not follow any fixed pattern in current practice. In some companies the inspector makes the plot, computes the values required and interprets the results. If the lot is deemed not acceptable, some person or committee of higher authority makes the decision between outright rejection and screening or salvage action. The inspectors are considered to be able to determine the type of distribution by comparison of the sample distribution with the 11 standards and to follow the appropriate procedure from that point. Some companies provide that, if he is unable to decide which standard type applies, he may refer the plot to a higher authority. In one company young ladies with no previous industrial experience and with less than a year's training in inspection were performing these functions to the satisfaction of the quality control department. They needed to refer only a small percentage of the daily plots to the supervisor for instruction for assistance.

The other extreme in inspector responsibility was represented by the case where the inspector simply recorded the readings in the order of occurrence and office personnel prepared the plots and made the computations and interpretations.

Between these two extremes many variations were observed. Opinions of the role the inspector can and should play varies considerably and appears to be based on the experience of the quality control supervisor, his opinion regarding the complexity of the plan, and his opinion regarding the opportunities for errors.

A number of simplifications in the duties of the inspector and the methods of computation were observed in several plants. These techniques and simplifications will be discussed later in this report.

2.4 The Administration of Lot Plot

In spite of some opinions voiced by individuals there appears to be no reason to assume that the administration of the Lot Plot is any more difficult than any other type of plan. In fact, there are certain aspects of Lot Plot which tend to simplify its administration. Since all plots begin with the random selection of fifty pieces and the preparation of the plot, there is no problem of selecting the correct plan to use for a particular lot size or AQL. After the plot is completed the remaining procedure for the determination of its acceptability is carefully prescribed according to the type of distribution obtained. Thus, the inspector can proceed without any instructions from his supervisor. Initial training in the Lot Plot procedures is required but probably does not take any more time than any of the other more elaborate plans. The procedure is so standardized that any reasonably intelligent person can develop considerable skill in the use of the plan in a short time. Errors in arithmetic and in plotting the points may be a problem here, just as in any variables plan.

The more difficult and more important part of the administrative problem is the determination of the products and quality characteristics to which the Lot Plot plan will be applied. Neither all the products nor all the characteristics of a single product will justify the use of Lot Plot. The selection of those characteristics for this kind of inspection must be based upon knowledge of the technical requirements and specifications, the production processes involved, quality history of the characteristics and the effects of accepting some defective items. Neither ordinary line inspectors nor inspection supervisors can be expected to make these decisions except in rare instances. Thus the Lot Plot becomes just one more plan available to the person charged with this responsibility and should not be looked upon as a cure-all for all incoming inspection.

One matter tends to complicate the administration of the plan. Mr. Shainin has pointed out the necessity for formally randomizing the sample. He insists upon the use of a table of random numbers or some similar method as a means of determining the specific pieces to be drawn for the sample. He has conducted experiments that show that failure to randomize will result in some bias about twenty-five percent of the time. Formal randomization requires more time and effort than "just picking one here and one there" for a sample. Even when the quality control department has given orders that samples will be randomized by use of a table of random numbers it is difficult to prevent line inspectors from taking the easy way, especially when work is piling up. Unless close supervision assures that the order is being followed in every case, there may be some doubt as to the validity of the values.

The writer observed a situation where an automatic pump was used to discharge a given volume of distilled water into a series of containers. A container was emptied into a graduate every thirty minutes to check on the accuracy of the pump. Examination of the records of this measurement for more than a week showed that the volume had not varied even one cubic centimeter from the specified 360 cc. Such accuracy was hardly to be expected. In another plant the hourly reading of a temperature gauge for the entire day had already been filled out before 9 A.M. If the validity of any sampling plan depends upon the constant use of some given technique or procedure, it is essential to provide enough supervision to detect any short cuts that may be tried.

2.5 Types of Distributions Encountered

The determination of the acceptability of a lot is largely dependent upon the type of distribution resulting from the plot of the sample of fifty pieces. Choosing one of the eleven standard types fixes the procedure to be followed in computing the lot limits. Serious errors in these estimates may result from the choice of the wrong distribution, and it is seldom that a sample distribution is as smooth or as nearly continuous as the standard distributions. Errors in this judgment are easily justified.

Each company visited was asked what types of distributions had been encountered in its experience. In those companies where the decision as to type was being made by persons with a substantial background in statistical quality control a fairly large number of the standards had been recognized. However, in those instances where line inspectors were making the decisions, the distributions observed were confined to only three or four types. All in all, the reports showed a preponderence of normal distributions (types 1 and 3) with bimodal (types 7 and 10) and skewed distributions following. It was rare that another type was reported and even more rare to find that a distinction had been made between 6 and 8, 3 and 9, or 2 and 11.

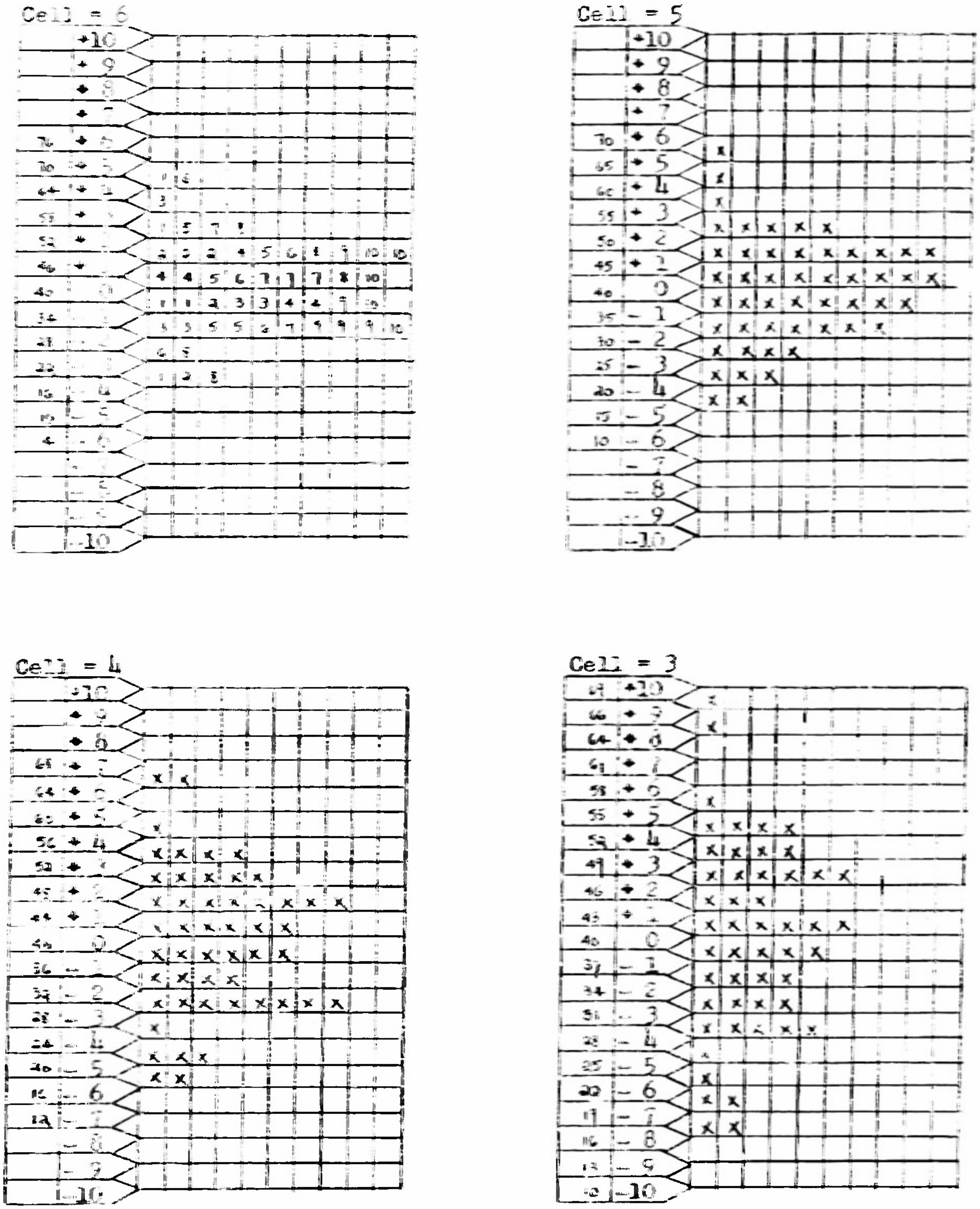
In some instances no attempt was made to classify the plot. The procedure for a normal distribution (type 1) was used in each case. Clearly, such a procedure eliminates one of the hazards of using the Lot Plot but introduces other hazards.

In at least one company no attempt was made to calculate lot limits. The specification limits were drawn on the Lot Plot form at the proper positions and if none of the parts fell outside the specification limits, the lot was assumed acceptable.

The distribution of a sample will take on radically different shapes as the cell interval is changed. The cell interval is determined by the range of the first subgroup of five pieces. According to the Lot Plot instructions a cell interval is selected and if twice the range (in those cells) of the first subgroup is between 7 and 16 (inclusive), the cell interval is adequate. Thus the cell interval may be first chosen as the finest division measurable by the instrument or some multiple of that division. A number of widely different cell intervals will meet this criterion in most cases. Furthermore, repeated sampling from a single universe will show a substantial variation in the range of the dispersion of the first subgroup. This can result in the cell interval being chosen unnecessarily small, or unsatisfactorily large. The extent of the effect of the cell width on the distribution cannot be predicted in advance.

Figure 2 shows the same sample plotted with four different cell

Figure 2.



intervals. The results are self-explanatory. There must be some doubt in the reader's mind as to which of the standard types these represent, and it is doubtful if he would choose the same type for all four if he encountered these plots individually.

2.6 Materials Review

The disposal of a lot is seldom the simple decision between acceptance and rejection. A great many factors enter into the problem of material procurement, and a lot that would be rejected without hesitation on one day may be eagerly accepted on another. Consequently, the disposition is usually either to accept or to submit the lot to a materials review committee which in turn may accept, reject, or order the lot to be screened. (Of course, rejection usually results in screening and the replacement of defective parts with good ones.)

When attribute acceptance plans are used the materials review committee must base its decision upon the meager information available from the original sample or cause a larger sample to be inspected. All that the committee knows is the per cent defective in the sample. It has no information regarding the variation in the lot. For this reason one of the greatest advantages claimed for the Lot Plot plan is derived from the additional information that is supplied to the review committee. Its decision then can be based upon the estimate of the limits of variation beyond the specifications and the probable per cent defective in the lot. The lot limits provide a basis for estimating the effects of random mating of these parts with others, and the computation of the probable number of misfits that will be encountered when the parts are assembled. The shape of the distribution of the sample pieces has been taken into consideration in the computations of the estimates, but the graphic presentation permits the committee to draw its own conclusions as to the best disposition.

Reports from the companies visited indicate that the assistance of the Lot Plot in materials review is highly appreciated. A number of companies reported that very satisfactory results had been obtained from its use in this respect. One Air Force Procurement officer verified these conclusions and stated that a large amount of material had to be accepted by the materials review procedure and that those lots for which Lot Plots were available were reviewed with considerably greater confidence that the best possible disposition was actually being determined.

Reliance on the Lot Plot for materials review purposes is based upon the assumption that the information is reasonably accurate. If the information supplied is erroneous, then the materials review board may be worse off than if the information were not available.*

Other variables sampling acceptance plans [5, 6, 7, 8] provide reliable estimates of the lot mean, standard deviation and probable limits. If desired, a frequency distribution could be plotted for any variables plan that would be just as useful as the Lot Plot. The same disposition of lots is

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*For a discussion of the reliability of the information from a statistical viewpoint, see Technical Report No. 18, "Some Theoretical Aspects of the Lot Plot Sampling Inspection Plan," by Lincoln E. Moses.

possible and the materials review procedures could be the same.

2.7 Strays

Stray pieces present a special problem in the Lot Plot. The stray is defined as a piece beyond the lot limits and type II distribution shows strays as single pieces separated from the main distribution by one or more cells. These pieces are used in computing the average range and the sample mean just as are all other pieces in the sample.

Strays may occur as a result of set-up pieces being included in the lot even though they differ substantially from the lot. They may also occur in the sample as the natural result of sampling from a lot that has a long tail or from a heavily skewed lot.

The estimates of the most likely number of strays in the lot and the number of cells outside the main distribution over which they may be expected are obtained from two charts provided in the Lot Plot plan. The estimate of the most likely number of strays is based upon General Simon's $I_q = 0.5$ chart for the inverse solution of the incomplete Beta function, and represents the number which will be exceeded as often as not. The second chart is based upon the assumption that the strays are part of a distribution with the same standard deviation as that for the sample of fifty pieces but with a different mean. A 0.90 probability limit is computed for ranges of samples of the size of the expected number of strays. Different limits were computed and charted for this purpose. Two of Wilks' theorems [4] provide the basis for this approach. These theorems state:

"94% of population values exceed the least value (or less than the greatest value) found, with probability of 0.95."

"90% of population values are included between the least and greatest values found, with probability of 0.95."*

It appears that this approach to the estimation of the probable number and limits of the strays is conservative, but users have not found this technique of extensive value. A small number of companies reported that attempts were made to use this technique when a type 11 distribution was recognized, and one company noted that it had been successful in stopping a supplier from including set-up pieces in shipments. However, it appears that very little use is being made of the charts for strays.

2.8 Screened Lots and Natural Cut-off Points.

It is recognized that many companies, in an effort to maintain a reputation for quality output, screen lots that are known to contain an excessive number of defective prior to shipment. The original lot may have been reasonably close to a normal distribution and the screening just removed a portion of the distribution beyond the specification limit. This condition is represented by Lot Plot types 6 and 9. In type 6, it appears that unnecessarily strict tolerances were established for the screening operation and some acceptable items were removed by the vendor. In type 9, it appears that the limits for screening were lower than the specification and some defective parts were shipped. These conditions

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*Quoted from "the Hamilton Standard Lot Plot Method of Acceptance Sampling by Variables," by Dorian Shainin, <u>Industrial Quality Control</u>, Vol. VII, No. 1, July, 1950, p. 27.

presumably can be true, but they may also result from either or both of two kinds of errors. First, the gauges of the producer may differ from those of the consumer by enough to make these discrepencies occur. Second, the random sampling procedure may simply fail to detect any of the smaller pieces present in the lot. Failure on the part of the vendor's inspectors to do a good job of screening may result in a few defectives remaining in the lot which show up as strays or as a type 9 distribution in the Lot Plot. This could also be a deliberate attempt to pass items that are near the specificiation limit.

Type 8, the half-normal distribution, may be mistaken for a screened lot of type 6. Many industrial processes result in the generation of only a portion of a normal distribution, called a half- or half-plus-normal distribution. In several different plants heat treating processes resulted in conditions like these. The Lot Plot proved to be a very valuable tool as a method of determining whether the treatment had been carried far enough or too far. It is worth noting, however, that in most of these plants, if the Lot Plot indicated that the process was not properly centered, the whole lot was re-treated. No attempt was made to screen acceptable from non-acceptable items because the cost of heat treating again was less than the cost of screening.

Concentricity, squareness, out-of-roundness, and other properties are frequently measured from a zero point with the majority of the items occurring near that point and decreasing in frequency farther away from it. Such situations practically always result in a half- or half-plus-normal distribution which may be mistaken for a type 6 or 9 unless the inspector who makes the Lot Plot recognizes and understands the cause. The proper disposition of the lot depends upon the distinction between a type 8 and types 6 or 9.

Some of the companies making the greatest use of Lot Plot methods stressed the importance of the inspector's having an adequate knowledge of production processes and distributions to be expected from the processes. This specialized knowledge is not essential for acceptance by attribute plans or by some variables plans.

3.0 Criticisms of Lot Plot

The Lot Plot method of acceptance sampling has been subject to considerable criticism, both direct and indirect, as well as to praise. The criticisms vary according to the needs of the users and the nature of the items to be accepted.

Mr. Shainin [3] has reported that criticisms reaching him had fallen into four groups:

1. The plan is too tight for most industries.
2. Its mathematical background is not sufficiently well established.
3. Several frequency histograms of fifty pieces each from the same lot will not show the same shape of distribution.
4. The staggered scale employed on the form is confusing.

These criticisms agree in substance with those reported by the companies visited in reference to this report.

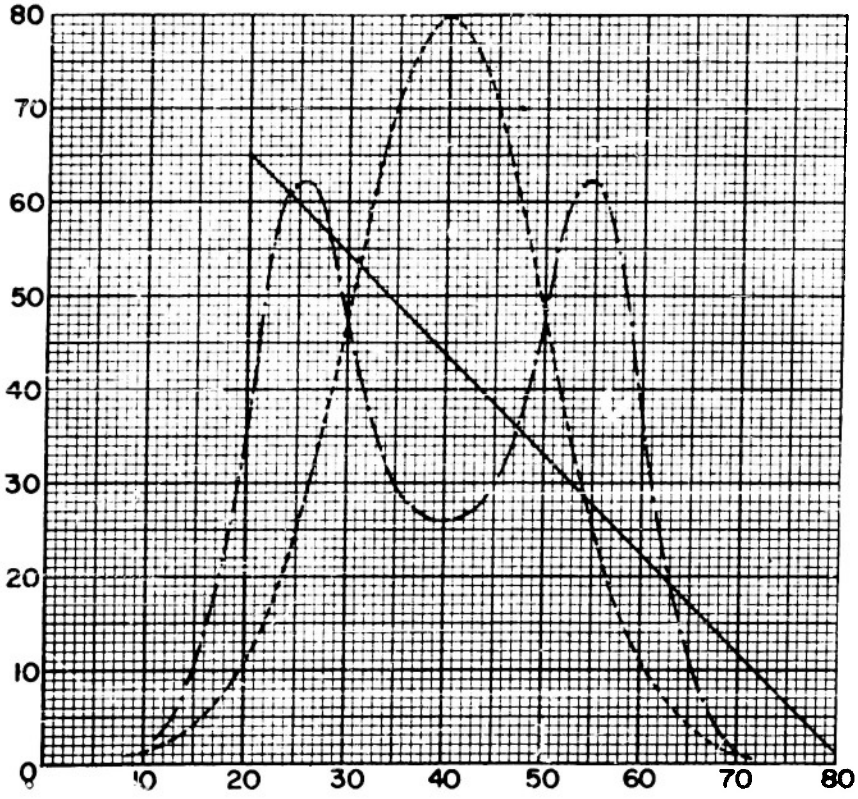
Mr. Shainin has defended the plan reasonably well in most of these areas. In reference to the first criticism, he pointed out that the object of the Lot Plot is to obtain as accurate a picture of the actual condition of the lot as "is humanly and practically possible." He admits that his O. C. Curve is very "tight", but explains that the disposition of the lot could be almost anything, depending upon the indicated condition and the current urgency for parts. It must be admitted that the establishment of AQL or LTPD values in advance of production is frequently arbitrary and not based upon known consequences of accepting lots of greater fraction defective. If an AQL of 2.00 is determined to be the quality that may be economically used with reference to any of the attribute plans the same can be assumed to be true of lots to be accepted by Lot Plot. That is, the lot may be accepted even though the lot limits exceed the specifications provided the estimated fraction defective is equal to or less than 2.00%. Of course, instructions to this effect must be issued prior to the sampling for the Lot Plot. If the Lot Plot fails to provide the necessary accuracy in the estimates of the fraction defective, then the entire criticism may be disregarded.

In regard to the second criticism, Mr. Shainin claims, and it seems intuitively correct, that the schemes for handling the unusual and irregular distributions are such that any errors are more likely to be in a conservative direction than otherwise. If this is true, then this only reinforces the first criticism, that the plans are too tight. The simplified methods of handling these irregular distributions, also, may warrant the risk involved, if these simplified practices tend to reduce the number of good lots that are unnecessarily 100% inspected as a result of the Plots. Some of these simplified practices will be discussed at greater length in paragraph 5.0.

Dr. Moses, in Technical Report No. 18, "Some Theoretical Aspects of the Lot Plot Method of Sampling Inspection," has investigated the mathematical reliability of the plan and the reader is urged to inspect that report for a treatment of the subject.

The third criticism is met by the same argument from Mr. Shainin, i.e., "The result from such samples from a single lot should, a very high percentage of the time, then be either so close to the lot conditions, or so that any error will be in a safe direction." To check this point, an experiment was tried in a Statistical Quality Control class at Stanford University.

Three chip distributions, each containing 2,000 chips, were prepared. (See Fig. 3). These represented a normal, a bimodal and a right triangular distribution. The mean of each of these distributions was 40 and the limits were maintained as nearly the same as was consistent with the other conditions. Each of nine students drew samples of fifty pieces from these distributions on two different occasions and did not know at anytime anything about the shape or nature of the distributions sampled. The samples of fifty were drawn by thoroughly mixing each distribution prior to each draw, but a table of random numbers was not used. Each student then prepared a Lot Plot, following very closely the instructions given in the Shainin article in Industrial Quality Control, and then classified each plot as one of the 11 standard types. The results of the experiment were as follows:



Distribution No. IV

Distribution No. V

Distribution No. VI ; $\bar{X}' = 40$, $\sigma' = 10$ for normal distribution

Figure 3

Types of sample distributions found from:

Type	Normal Distribution	Bimodal Distribution	Triangular Distribution
1 normal	6		1
2 normal	4		
3 normal	2		
4 truncated			1
5 skewed	2		3
6 half-plus	1	1	2
7 bimodal		4	
8 screened		1	3
9 bimodal			
10 bimodal	2	9	10
11 strays	1	2	2
Some odd distribution not covered by standards		2	2

Thus 6 of the 18 plots (33 1/3%) from the normal distribution were interpreted as being something other than normal and would result in substantial errors in conclusions drawn from the standard Lot Plot procedure, even though the errors would in this case be in the conservative direction.

From the bimodal distribution, 5 of the 18 plots were interpreted as being something other than a bimodal distribution. (28% in error). An examination of Figure 3 shows that the original distribution was a very pronounced bimodal with the modes separated by a substantial distance.

If types 6 and 8 are assumed to be the expected distributions from a triangular parent distribution, then only 7 of the 18 plots can be assumed to have represented the parent universe. All other types would result in errors of estimation of the lot limits. (61% in error.)

From this short experiment it seems clear that distributions of samples of 50 from the same lot can be sufficiently different to cast grave doubts upon the reliability of the method. This may be expressed another way. Repeated sampling from the same lot by the Lot Plot method will not result in consistent distribution shapes. The statistical precision of the method is poor.

The fourth criticism, that the staggered scale is confusing, is of so little importance that it is hardly worth mentioning. Anyone can design the form to suit his own preferences.

Other criticisms uncovered in the plant visits included the following:

1. There are too many types of distributions without clearcut lines of distinction between the similar types. Attempts to use these different types frequently lead to confusion and excessive caution in determining the disposition of the lot.

2. The procedures are unduly complicated and sufficiently good results can be obtained by treating all lots as normal and computing the lot limits and the per cent defective. In other words, the plan should be simplified.

3. Some companies did not allow the line inspectors to make any computations or decisions regarding the Lot Plot, because it was felt

that adequate training would be excessively costly. Thus, more work was added to the load for the office clerical force.

4. Screening of a large number of rejected lots consistently failed to show as many defectives as the Lot Plot procedures indicated for the lots. Thus more lots were screened than should have been. This is, of course, a conservative practice, but may not be justifiable in terms of the cost of acceptance.

5. It was felt in some companies that it is dangerous to accept very large lots (25,000 to 70,000) on the basis of a sample of only fifty pieces. At least one company followed the practice of breaking large lots into two or more smaller lots (never less than 10,000) and took a sample of fifty from each of the sub-lots. Then, if all plots for the sub-lots showed acceptable quality, the whole lot was accepted. Otherwise, the lot was held for review by the salvage committee.

4.0 A Controlled Experiment to Evaluate Lot Plot

In April, 1952, The Aircraft Industries Association, by a bulletin IC No. 52-21, invited certain members to participate in a "Test to determine the relative effectiveness of 100% inspection, Mil-Std-105A, and Lot Plot Inspection." The plan called for those companies who wished to participate to set up a procedure within the framework of the bulletin whereby lots would be inspected by Lot Plot, Mil-Std-105A and 100% inspection, without the inspectors employed on each of these plans knowing that other inspectors would inspect the same lots by other methods. The results were to be recorded and reported to the Inspection Committee of the AIA. Where feasible, the lots were to be 100% inspected two or three times after the initial 100% inspection to verify the actual condition of the lot.

A Mil-Std-105A plan was selected to match the O. C. Curve that had been published for the Lot Plot method. This resulted in the selection of an AQL of 0.035 with a sample size of 750 pieces, randomly chosen, and an acceptance number of one. The company had the choice of using the corresponding double and sequential sampling plans. The user was also cautioned to use a set of random numbers as a means of selecting the sample for the Lot Plot, and a set of random numbers was included in the bulletin.

The results of this controlled experiment were reported in bulletin IC No. 52-47, November 18, 1952. A total of 112 lots were reported to have been inspected under the plan in such a way that the results could be used. Other reports had to be omitted for various reasons. The lot sizes ranged from 70 to 2721 and the actual fraction defective, as determined by "several" 100% inspections ranged from zero to about 89%. Only 8 lots were accepted by the Lot Plot and the author of the report stated that at least four of these Lot Plots were made without the benefit of a sample selected by means of random number tables. The author of that report seems to feel that these lots should not have been accepted by the Lot Plot since each lot contained at least one defective. It is an accepted principle that no sampling plan will protect the consumer from the acceptance of some defective product. It seems significant that the Lot Plot method rejected and held for materials review lots later determined to contain the following percentages:

Lots Rejected by Lot Plot

Actual Fraction Defective	Estimated by Lot Plot % Defective
1.43%	2.0%
0.0	3.5
0.57	1.0
1.24	2.0
0.62	1.7
0.25	0.8
0.5	0.5
0.5	0.2
0.64	3.0
1.4	1.0
0.39	0.7
1.37	1.4
0.23	0.2
0.0	0.2
0.66	1.0
0.013	1.0
0.8	1.0

The Lot Plot method over estimated the fraction defective in 61 cases and under estimated it in 46 cases. Figure 4 shows the distribution of the errors in estimation, relative to the magnitude of the error. All of these data reenforce the idea that the Lot Plot is a "tight" plan and that it is conservative.

The AIA report states that the ineffectiveness of the Mil-Std-105A and 100% inspection plans is indicated by the large number of defective items missed in these two inspections. The results from these two inspections and the following 200% and 300% inspection are herewith recorded:

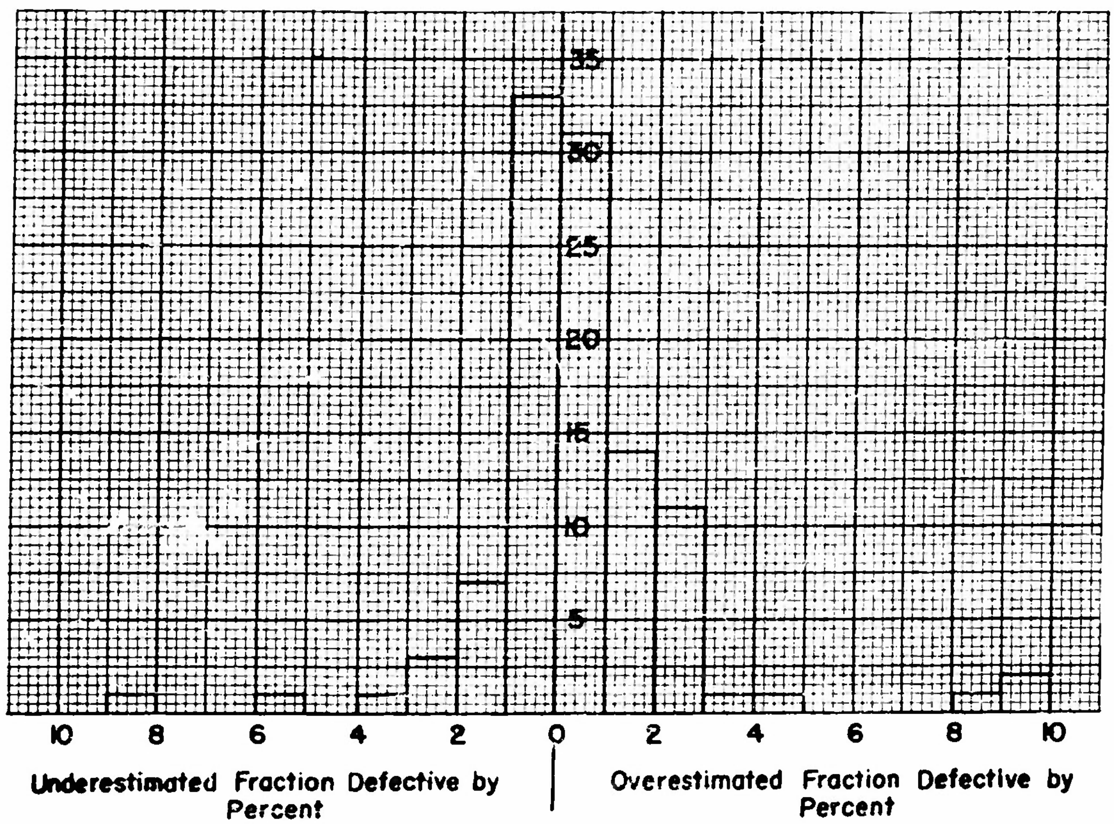

35
30
25
20
15
10
5
10 8 6 4 2 0 2 4 6 8 10
Underestimated Fraction Defective by Percent
Overestimated Fraction Defective by Percent

Figure 4

Defectives found in Lots Accepted by Mil-Std-105A

From Mil-Std-105A Inspection	100% Inspection	Several 100% Inspections	Lot Size
0	0	68*	500
0	0	136*	500
0	0	7	500
0	0	41*	500
0	1	55*	520
0	0	130*	520
0	0	25	520
0	0	46*	611
0	0	137*	618
0	0	103*	618
0	0	28	1,000
0	0	95*	1,197
0	0	2	1,500
0	0	110*	1,777
0	0	47*	1,777
0	4	306*	1,800
0	0	49*	1,938
0	0	44*	2,000
0	0	16	2,000
0	0	1,506*	2,126
0	0	88*	2,500
0	0	344*	2,500

The above lists all of the lots accepted by the Mil-Std-105A plan for which the final inspection showed more than one defective in the entire lot. The difference between the number of defectives found in the sample and in the first 100% inspection and the number found by several 100% inspections could not possibly have happened in those cases marked with an asterisk if the quality of the inspection had been the same in all cases. It seems quite obvious that the inspection of the initial sample for Mil-Std acceptance and for the first 100% inspection (where that differed from the sample inspection) was performed in a careless manner or else the inspector was not properly trained and did not know what to look for. If all the inspections had been performed on each lot by the same inspector, such differences would not likely have occurred. Perhaps the use of different inspectors and the repeated inspections is more important as an indicator of the difference between inspectors than as an indicator of the effectiveness of the three methods under study.

If the lots denoted by the asterisk above are considered to have been accepted as a result of inferior inspection rather than as a result of any failure on the part of the method, then it can be said that the Mil-Std-105A plan accepted only 5 lots containing more than 2 defective out of the 112 presented for acceptance. This is quite different from the conclusion of the AIA report which stated that a total of 57 lots contained "defective material." The report's conclusion that the Lot Plot provides the greatest protection against acceptance of defective material seems to be founded only upon the fact that the plan is conservative and rejected most of the lots, even those lots containing very low fractions defective. Had the inspection been equally good in all cases the results probably would have been substantially different.

5.0 Variations From Lot Plot Procedures

5.1 Variations in Forms

The Lot Plot form, as originally devised, contained four major sections: the heading which identified the Vendor, part, date, quantity, etc.; the main body of the form where the distribution of the sample was plotted; the right side where spaces were provided for the computation of the average range, mean, and lot limits; and the lower section where the disposition of the lot was indicated and the extent of variation from the specifications stated (if any). Later, Mr. Shainin devised a simplified scheme for computing the mean which eliminated the necessity for columns for the sum and average of the observations in each subgroup.

Examination of Lot Plot forms from a number of companies reveals that few companies use the original form. Most have modified the forms to accomplish certain objectives. The staggered spacing of the cell boundaries relative to the spaces for plotting the observations has been abandoned by many. This change seems to be of little importance. The greatest changes have been made in the computation section on the right side of the form. Those companies that have gone to a simplified means of computing the means have in general omitted this section and replaced it with a check list of recommendations to the vendor. One used this section as a space for recording the results of acceptance sampling of previous lots. Another company used this space for $\bar{X}$ and R charts as a record of successive lots from one vendor. In both of these cases one Lot Plot form is used as the initial acceptance form and as a record of up to twenty lots following it. The objective is to present a formalized quality history of the part from each vendor, but other methods appear to be superior to these.

Some companies have found it advantageous to provide more specific instructions in regard to the computations and have revised this section to include detailed, step-by-step instructions. These instructions are usually given in words rather than as formulas.

The disposition section of the form has been changed by some companies to accommodate specific problems arising from the material review procedures used. This is particularly true of those companies that are not dealing with government agencies. Those companies that never use an attribute sample following the Lot Plot have largely eliminated the space for additional sample data.

5.2 Use of frequency distributions only

A number of companies, as a result of experience with the Lot Plot, have resorted to the use of frequency distributions only, without any calculations of lot limits, for acceptance purposes. Basically, the frequency distribution is compared with the specifications, and if the distribution is centered and well within the specification limits, the lot is accepted. In one plant, where certain small parts are received daily in lots of 25,000 to 50,000, a sample of 25 pieces was drawn from the lot and plotted. Acceptance was based upon the criteria mentioned above. It should be understood that these parts had to be adjusted by hand as they were assembled, and a defective part was easily recognized during this adjustment and assembly. Furthermore, these parts were inexpensive, and the good relations between the vendor and consumer depended, to some extent, upon the reimbursement by the vendor when an appreciable number of parts were found to be defective. A quality history, by lots, was maintained by the customer, who informed the vendor whenever it appeared that the quality of the product had changed.

Other companies followed similar procedures except that the sample size was usually at least fifty pieces and sometimes as many as one hundred. Also, some companies followed this procedure, with a sample size of fifty for acceptance, but computed lot limits and the estimated percent defective if either tail of the distribution extended beyond the specification limit.

An additional sample of 25 pieces was drawn from the lot in one instance if the plot of fifty pieces did not give a reasonably conclusive shape to the distribution. Otherwise, the procedures were the same.

Another company, dealing in small parts, prepared a tray with small compartments which corresponded to the blocks on the Lot Plot form. As the fifty pieces were inspected, they were distributed to these small compartments. Then the Lot Plot form was completed by placing check marks in the corresponding blocks. No attempt was made to separate the fifty pieces into subgroups of five each. The standard deviation was later computed in the office using the common method for grouped data, and the lot limits were estimated by the ordinary statistical procedures. No attempt was made to classify the distributions or follow the prescribed procedures.

5.3 Computation procedures

The previous paragraph reported a case where common statistical procedures were used for the computation of the standard deviation of the sample and the lot limits. One company improved on this method by preparing two transparent overlays, one for FX values and one for FX^2 values. The clerk placed the FX overlay over the distribution and using a comptometer determined the mean of the distribution. Then placing the FX^2 overlay so that its zero line was on the mean of the distribution, the clerk obtained the sum of the FX^2 values. From a previously prepared table of FX^2 and 3σ values, the 3σ limits in terms of cells were obtained.

One secretary performed these computations for about 180 Lot Plots per day in addition to performing certain other duties. Each Lot Plot required less than 2 minutes by this method.*

6.0 Similar Plans

A number of persons are interested in and attempting to devise new plans for acceptance sampling that will make use of the better points of the Lot Plot plan and avoid some if its disadvantages. Very little has been published at this time regarding such plans.

Mr. David A. Hill, of Hughes Aircraft Company, however, has published the details of his proposed plan. The plan is called "The Lot Template Method of Inspection by Variables" [1]. The plan begins with the random selection of a sample of fifty pieces from the lot. These pieces are measured and a frequency distribution plotted without regard to subgroups. As in Lot Plot, the cell interval must be selected before plotting is started. This plan suggests that the range of the first seven pieces be compared with a table provided in the plan and the units per cell determined from the table. It is desirable for the plot to contain from nine to fifteen cells.

Transparent templates for normal distributions of fifty pieces with base widths of from 5 to 18 cells and for skewed distributions (k = 1 and k = 2) with the same base widths are prepared in advance. On each

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*This method and cuts of the overlays were published in Industrial Quality Control under the title of "A Convenient Short Cut in the Use of Lot Plot," by Richard Wilson, Vol. VIII, No. 5, March, 1952, pp. 32-33.

of these templates, the upper and lower limit of occurrences in each cell is indicated. These limits are based upon a probability of 0.95 that the observations in each cell of a sample distribution drawn from a normal distribution will fall within these limits.

The Lot Templates are placed over the sample distribution in succession and the smallest one that will contain all the observations within the cell limits is selected. Since the base width of the template depends upon the standard deviation expressed as a function of the cells (i.e., $\sigma = 2.5$ cells) the template selected estimates the standard deviation of the distribution in cells.

The plan further provides that Acceptance Limits will be drawn on the Lot Templates to provide a given AQL for the acceptance plan. Then, when the specification limits are drawn on the plot, the lot is accepted if the Acceptance Limits fall within the specification limits. The illustrations provided in the presentation of this plan showed only one set of Acceptance Limits on each Lot Template, but different Acceptance Limits representing different AQL's could be drawn on each. Doing so would reduce the number of Templates needed if different AQL's were commonly employed by the user.

Templates are not commercially available, but the publication from Hughes Aircraft contains all the information necessary to plot the templates for both the normal and skewed conditions and for determining the location of the Acceptance Limits and the Cell Limits.

Expressions for the theoretical O. C. Curve for the normal and skewed conditions were derived, but it was recognized that a number of types of errors could make an appreciable difference between the actual and the theoretical O. C. curves. Therefore, a large number of drawings of fifty pieces from a Shewhart Bowl were made and the actual O. C. curves for certain lot conditions determined. The author pointed out the fact that the O. C. curves will differ with the lot conditions but showed a number in comparison with attribute sampling plan O. C. curves to illustrate the superior efficiency of the Lot Template method over the attribute plans with similar AQL values.

This plan appears to present a number of features that will appeal to industry. It provides for the selection of an AQL to suit the needs of the situation. The use of the template eliminates most of the computations associated with the Lot Plot. There is only one basic procedure to know and use. The Templates, once prepared by a qualified person, can be used by inspectors with very little training. It also has some disadvantages. A number of Templates will ordinarily be tried before the smallest one that includes all the observations will be found. Each inspector must have a full set of Templates and attempts to reduce the size of the set will undoubtedly increase the possibilities for errors.

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